

## Proof VS-1

Accepted

Not Accepted

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

- First due date **Thursday, November 6**.
- Turn in your work on a separate sheet of paper with this page stapled in front.
- Do not include scratch work in your submission.
- There is to be **no collaboration** on any aspect of developing and presenting your proof. Your only resources are: you, the course textbook, me, and pertinent discussions that occur **during class**.
- Follow the Writing Guidelines of the Grading Rubric.  
([http://math.ups.edu/~bryans/Current/Fall\\_2008/290inf\\_Fall2008.html#tth\\_sEc5.1](http://math.ups.edu/~bryans/Current/Fall_2008/290inf_Fall2008.html#tth_sEc5.1))
- Retry: Only use material from the relevant section or earlier.
- Retry: Start over using a new sheet of paper.
- Retry: Restaple with new attempts first and this page on top.

*“By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and, in effect, increases the mental power of the race.”* – Alfred North Whitehead

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 VS-1 (Section S)

1. Use the Principle of Mathematical Induction to prove the following theorem.

**Theorem 1** *If  $W_1, W_2, \dots, W_n$  are subspaces of a vector space  $V$ , then their intersection  $\bigcap_{k=1}^n W_k$  is also a subspace of  $V$ .*

2. Show that no analogous theorem can be true for unions by specifying two particular subspaces of  $\mathbf{C}^3$  whose union is not a subspace of  $\mathbf{C}^3$ . Be sure to explain why the union is not a subspace.

**Notes:**

- The intersection of sets  $S$  and  $T$  is defined by  $S \cap T = \{x : x \in S \text{ and } x \in T\}$ .
- This extends naturally to the intersection of a finite collections of sets  $S_1, S_2, \dots, S_n$  with the definition

$$\bigcap_{k=1}^n S_k = \{x : x \in S_k, 1 \leq k \leq n\}.$$

- The union of sets  $S$  and  $T$  is defined by  $S \cup T = \{x : x \in S \text{ or } x \in T \text{ (or both)}\}$
- The easiest subspaces to look at are those that are the spans of sets of vectors.